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Chaire Desjardins en finance responsable

par

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The Development of a Canadian Capital Market for Longevity-linked Securities

CAHIER DE RECHERCHE

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Préambule

La gestion financière responsable vise la maximisation de la richesse relative au risque dans le respect du bien commun des diverses parties prenantes, actuelles et futures, tant de l'entreprise que de l'économie en général. Bien que ce concept ne soit pas en contradiction avec la définition de la théorie financière moderne, les applications qui en découlent exigent un comportement à la fois financièrement et socialement responsable. La gestion responsable des risques financiers, le cadre réglementaire et les mécanismes de saine gouvernance doivent pallier aux lacunes d'un système parfois trop permissif et naïf à l'égard des actions des intervenants de la libre entreprise.

Or, certaines pratiques de l'industrie de la finance et de dirigeants d'entreprises ont été sévèrement critiquées depuis le début des années 2000. De la bulle technologique (2000) jusqu'à la mise en lumière de crimes financiers [Enron (2001) et Worldcom (2002)], en passant par la mauvaise évaluation des titres toxiques lors de la crise des subprimes (2007), la fragilité du secteur financier américain (2008) et le lourd endettement de certains pays souverains, la dernière décennie a été marquée par plusieurs événements qui font ressortir plusieurs éléments inadéquats de la gestion financière. Une gestion de risque plus responsable, une meilleure compréhension des comportements des gestionnaires, des modèles d'évaluation plus performants et complets intégrant des critères extra-financiers, l'établissement d'un cadre réglementaire axé sur la pérennité du bien commun d'une société constituent autant de pistes de solution auxquels doivent s'intéresser tant les académiciens que les professionnels de l'industrie. C'est en mettant à contribution tant le savoir scientifique et pratique que nous pourrons faire passer la finance responsable d'un positionnement en périphérie de la finance fondamentale à une place plus centrale. Le développement des connaissances en finance responsable est au cœur de la mission et des intérêts de recherche des membres tant du Groupe de Recherche en Finance Appliquée (GReFA) de l'Université de Sherbrooke que de la Chaire Desjardins en finance responsable.

La finance responsable (ou durable) vise donc notamment à développer des modèles, des produits et des services ainsi qu'à orienter les marchés financiers et les décisions en matière de fiscalité dans une perspective durable et responsable. À cet effet, les Professeur(e)s Frank Coggins, Claudia Champagne et Lyne Latulippe ont publié en 2018 aux Éditions *Thompson Reuters* un recueil de textes s'intitulant « Éléments de la finance responsable : une approche multidimensionnelle ». Ce collectif contribue à mieux définir et délimiter la finance responsable en la décloisonnant dans une perspective multidimensionnelle. Il regroupe des textes d'universitaires de différentes disciplines ainsi que de spécialistes de l'industrie financière, propose des pistes pour tendre vers une meilleure finance, vers une finance plus responsable. Le présent cahier de recherche constitue l'un des textes (chapitres) tirés de ce collectif.

The Development of a Canadian Capital Market for Longevity-linked Securities

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The improvement of the human lifespan has been observed for over a century, and the causes are well documented. Longevity risk, however, is relatively new outside of the insurance industry and has until recently been largely ignored by pension plan sponsors, governments and individuals whose focus was primarily on the management of investment risks. Low interest rates, the aging population and the anemic growth prospects in most developed economies, and the lower expected investment return they entail have contributed to a greater awareness of the longevity risk and the need for a wider range of solutions through financial markets.

In this article, we first describe longevity risk and the rationale for developing effective and accessible hedging solutions to what has essentially, and until recently, been regarded by most as medical and societal prowess. In the second section, we present two longevity-linked securities that can be used as hedging instruments. It is followed by an overview of the data and the modeling methods applied to construct longevity scenarios and test the hedge effectiveness of the instruments. In the next section, we present and comment on the results of our tests. We finally conclude with a brief discussion on future work and challenges ahead.

1. THE LONGEVITY RISK AND WHY BOTHER

Longevity risk is the danger that we outlive our retirement savings and the ensuing challenge to maintain the living standard to which we are accustomed or to afford ever increasing health care and retirement housing costs. Longevity risk also affects all organizations, private or public, which guarantee a lifetime income, and the reimbursement of long term health care or other social services. As such, individuals, pension plans, insurers and taxpayers may be adversely affected by the improvements of survival rates.

Traditionally, the longevity risk, along with investment risk, was transferred from individuals to private or public defined benefit pension plans. Longevity risk differs from the investment risk as it is not diversifiable. Private plan sponsors wishing to manage their longevity risk will transfer it to insurers through annuity contracts or complex bespoke longevity swap arrangements. And they increasingly rely on such contracts to reduce their risk exposure. For example, annuity purchase activity from Canadian private pension plans expanded from $1 billion in 2012 to almost $2.7 billion in 2016 (Willis Towers Watson, 2017). This trend is expected to accelerate as interest rates increase, since it will improve the financial position of defined benefit pension plans and at the same time it will lower the costs of buying annuities. This trend has already been observed in the United States and North America but it lags the U.K. in terms of investment and longevity de-risking activity.

Insurers and reinsurers have limited capacity to absorb a large amount of longevity risk at once despite their ability to partially offset it through their exposure to the risk of early death within their life insurance business. In Canada alone, assets held by employer-based pension plans and individuals through their retirement savings accounts amounted to over $3.1 billion at the end of 2015 (Statistics Canada, 2016). This represents a substantial imbalance relative to the capacity of the Canadian insurance industry to absorb the underlying liabilities should the trend towards purchasing annuities accelerate in the future.

The regulations that pertain to the insurance industry are under review and as regulators lean towards a greater harmonization globally, we can expect they will promote the use of internal models to manage longevity and mortality risks more actively. The incentive for doing so could include the recognition for such efforts through capital relief, a better appreciation of the risks and more appropriate pricing of life products. As plan sponsors and insurers turn their attention to longevity risk, solutions incorporating the broader financial market will be essential and will benefit all stakeholders by introducing a more efficient pricing mechanism of longevity and mortality risks.

The appeal of longevity-linked products to investors who are normally not exposed to mortality or longevity risks should stem from the longevity risk premium paid and the low correlation of this premium with any other market risk premium.

We review two basic longevity-linked derivatives, the survivor forward (S-Forward) and the survivor caplet (S-Caplet) which are the building blocks for the structuring of the two hedging instruments presented here: the survivor swap (S-Swap) and the survivor cap (S-Cap).

2. S-FORWARD, S-SWAP

The Life & Longevity Markets Association website[1] has sample termsheets of the longevity-linked instruments presented here.

The S-Forward is an agreement by which the buyer, or longevity hedger, will receive a variable amount based on the realized future value of a survivor index at maturity, in exchange for paying a fixed amount based on the risk-adjusted expectation, at the inception of the contract, of the survivor index at maturity. These two cash flows exchanged at maturity are the only payments made under the plain vanilla S-Forward.

An example of a survivor index would be the cohort of Canadian females who will be 65 years old on June 30th, 2017. The Canadian Human Mortality (CHMD) database has Canadian data going back to 1921.[2] More on this in the next section.

A life annuity issued by an insurer (in the wider sense of the term as it includes defined benefit pension plans) is a contract which pays the annuitant a fixed amount (e.g. $1) at the end of each year that he/she is alive. Therefore, the hedger of a portfolio of annuity contracts has to face a sequence of yearly liability payments based on the survival rate of the annuitants at the different year-end dates. As such, the hedger will typically enter into a sequence of S-Forwards with maturities matching the liability payments. It is natural to call this series of S-Forwards on the same index an S-Swap. Note that the fixed rates in this swap are not constant but, as survivor rates, they decrease with time unlike the constant fixed rate of an interest rate swap.

An insured (or bespoke) swap is an S-Swap whose payments are derived from the survival rates (realized and expected) of the actual population of annuitants. This instrument has the advantage of leaving no population basis risk to the hedger. But, like all bespoke

1. *A framework for pricing longevity exposure*, Life & Longevity Markets Association, online: <www.llma.org>.
2. <http://www.bdlc.umontreal.ca/chmd/>.

instruments, it is costly and it is even costlier to exit before the final maturity. This is a market for specialists (insurance and re-insurance companies) since it requires micro modelling of the survival rates for particular pools of annuitants.

The cohort based longevity linked instruments are more transparent since they rely on macro longevity tendencies in national populations whose statistics are maintained by (neutral) third parties like Statistics Canada and CHMD. We will revisit this topic in the next section.

3. S-CAPLET, S-CAP

The payment of an S-Caplet is determined by the same variables as an S-Forward. However, the hedger will receive the difference between the realized survival rate of the cohort and its expected survival rate only if it is positive. This has two implications: 1) as before, the hedger is compensated if survival rates are higher than expected; and 2) the hedger can keep the upside of experiencing lower than expected survival rates. This is equivalent to a call option on the survival rate of the cohort. Since this option position has no downside, the buyer of the option pays a premium to the seller at the inception of the transaction. This upfront premium incorporates the risk premium so there is no need to risk-adjust the expected survival rate.

As before, hedgers of annuity-like products want a string of these caplets with maturities matching those of their liabilities. A series of yearly S-Caplets on the same index is called an S-Cap. Again, the fixed rates on the caplets are decreasing unlike interest rate caps which have a fixed rate for all maturities.

4. MORTALITY DATA AND MODELING

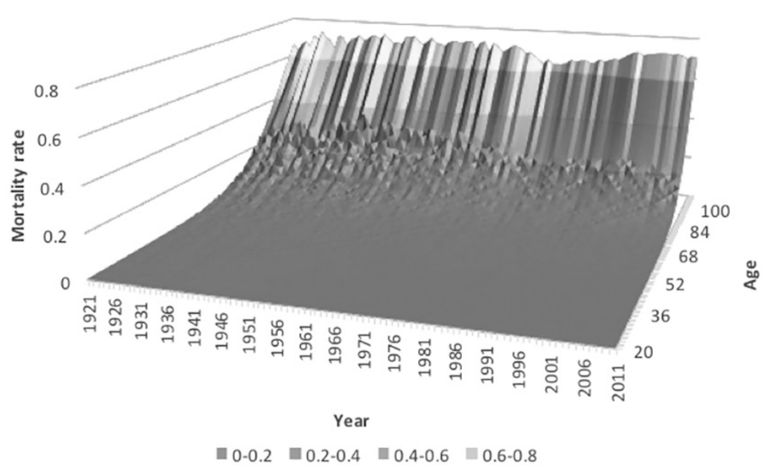
Annuity contracts and bespoke longevity swaps are based on the expected survival rates of the annuitants and determined at the origination of such contracts. In order to access a much larger capital base and foster the liquidity required by market based solutions, it will be paramount to have standardized indices form part of a transparent pricing methodology. The development of a large market hinges precisely on the competing requirements of devising a restricted set of indexes serving as a reference to a small set of liquid instruments, and the availability of indexes and instruments that are varied enough to form solutions that do not entail too much population basis risk, i.e. the risk of hedging one population's survival with the experience measured with a broader or different population.

In modeling mortality rates and changes to these mortality rates over time, we mostly focused on the crude central mortality rates for the overall Canadian population of all ages from 0 to 110, reported separately for women and men. We used the central mortality rates published by the Canadian Human Mortality Database (CHMD) from 1921 to 2011.[3] CHMD is affiliated with the Human Mortality Database (HMD), which collects national data for 37 countries or regions. HMD is a joint venture between the demography department of the University of California and the Max Planck Institute in Germany.[4] The mortality rates published by the CHMD are updated annually and are used by a variety of organizations. We determined that these rates were appropriate for the purpose of the study.

We denote the crude central mortality rate for a cohort aged x in calendar year t by $\mathbf{m}_x(t)$. It is defined as the ratio whose numerator is the number of people aged x who die in calendar year t; and whose denominator is the number of people aged x on June 30th of calendar year t.

The graph below illustrates the central mortality rates observed for Canadian women aged 20 to 110 over the 1921 – 2011 period.[5]

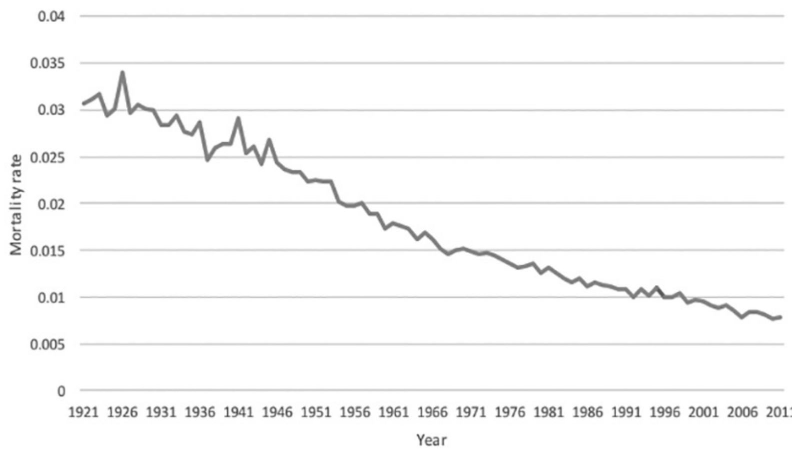
Mortality Rates for 20-110 Year Old Women



3. <http://www.bdlc.umontreal.ca/chmd/>.
4. <http://www.mortality.org/>.
5. <http://www.mortality.org/>.

The decline of mortality rates is a heavy trend, but changes in mortality rates are not constant from one year to the next and vary by age and gender. The impressive reduction of mortality rates is more apparent in the next graph, which shows the improvement of mortality rates for women aged 65 over the same period.

Moratlity Rates for 65 Year Old Women



Following a review of the literature dedicated to the modeling of mortality rates, we selected two financial models presented in the following articles by Wills-Sherris (2011) and Fung *et al.* (2015). The models were fitted to the mortality data in order to simulate the mortality improvements over time. The quality of fit was successfully tested for each model and the results were also validated by comparing them to those obtained from a standard time series model used by insurers for scenario analysis.

We are not going to delve too much into the models, but we will use some elements of the Wills-Sherris model[6] to introduce and link the different concepts used in the next section. Readers interested in the details of what follows should read the two Master Theses of Fatima Boutaggount[7] and Frédéric Fontaine.[8]

6. S. WILLS and M. SHERRIS (2011), *Integrating Financial and Demographic Longevity Risk Models: An Australian Model for Financial Applications*, Research Paper No. 2008ACTL05, UNSW Australian School of Business.
7. F. BOUTAGGOUNT (2017), *Modélisation et gestion du risque de longévité : Application à des instruments financiers liés à la longévité*, Université de Sherbrooke.
8. F. FONTAINE (2017), *Quelques résultats sur la modélisation et la couverture de portefeuilles exposés au risque de longévité*, Mémoire, Université de Sherbrooke.

The Wills-Sherris model is of Black-Scholes-Merton type, which is the best known model of financial literature. We will want to follow a cohort year after year so we will first make a slight modification to the central mortality rate in order to have a convenient notation to do so. The year t will now refer to the t^{th} year after an anchor/initial calendar year, for example 1921 + t.

We then define $m_x(t) = \mathbf{m}_{x+t}(1921 + t)$.

The Wills-Sherris (2011) model is then specified by this initial condition and by the following continuous dynamics:

$$dm_x(t) = m_x(t)[a(x + t) + b]dt + m_x(t)\sigma dW_x(t),$$

where (a, b, σ) are constant and $W_x(t)$ is a Brownian motion.

Since our financial instruments all have yearly payments, we work essentially with the discrete annualized form of the above equation:

$$m_x(t + 1) = m_x(t) + \Delta m_x(t) \quad (*)$$

where $\Delta m_x(t) = m_x(t)[a(x + t) + b] + m_x(t)\sigma \Delta W_x(t)$, and for each x,

$\Delta W_x(t) = W_x(t + 1) - W_x(t)$ are independent standard normal random variables.

We first calculate the parameters $(\hat{a}, \hat{b}, \hat{\sigma})$ that minimize the log-likelihood function over different periods ending in 2011. Then starting with the mortality rates in the first calendar year of such period, we use (*) in expectation (i.e. without the random term ΔW_x whose expectation is 0) to successively obtain mortality rates from the model. For each period, we then compute the Mean Square Error (MSE), which is the mean of the squared differences between the historical mortality rates and those obtained from the model. The period which produced the lowest MSE was the 37-year period between 1975 and 2011. We performed several statistical tests on the residual errors to ensure they were independently and identically distributed according to a normal distribution. We tested the robustness of the model by comparing projected mortality improvement rates with out-of-sample observations. The interested reader will find the details in Boutaggount (2017) and Fontaine (2017).

We are now ready to develop mortality forecast scenarios to test the efficiency of the S-Swap and S-Cap hedges. For a given age *x*, the mortality forecast process starts from the most recent mortality rate, in this particular case, the central mortality rate anchor was the 2011 observed mortality rate, i.e. $m_x(t) = \mathbf{m}_{x+t}(2011 + t)$. The model parameters $(\hat{a}, \hat{b}, \hat{\sigma})$ were determined based on the mortality data in the prior 37 years (i.e. from 1975 to 2011). This period should adequately reflect the mortality trend observed in the past, while being relevant to produce mortality rate forecasts for the next 20 to 30 years.

Once we have the central mortality rate forecasts, $m_x(t)$, the relationship between the central death rate at age *x* and the probability, $p_x(t)$, that an individual aged *x* will survive until age x + 1, can be written in the following form:

$$p_x(t) \approx \frac{1 - 0.5 * m_x(t)}{1 + 0.5 * m_x(t)}$$

From this expression, the probability that an individual in a cohort aged *x* will survive T years, denoted by S_x^T is expressed as:

$$S_x^T = \Pi_{i=0}^{T-1} \; p_{x+i}(i)$$

One nice feature of the financial models used here is the possibility to risk-adjust their trend by calibrating them to market prices. The effect of the risk adjustment on survival rates is to inflate them to account for the risk premium. In an S-Forward for instance, the hedger will pay a fixed rate based on the expected historical survival rate of the index plus a risk premium that compensates the hedge provider for the variability of his/her payment since it is based on the realized survival rate. To our knowledge, there is only one pricing data source in the literature; it is the 2004 longevity bond issuance of European Investment Bank in partnership with BNP Paribas and Partner Re.[9] We will return to this point.

We performed this calibration and all rates obtained through this risk-adjusted methodology are labelled $\tilde{m}_x(t)$, $\tilde{p}_x(t)$ and $\tilde{S}_x^T$.

9. D. BLAKE, A. CAIRNS, G. COUGHLAN, K. DOWD and R. MacMINN (2013), "The New Life Market", 80(3) *The Journal of Risk and Insurance* 501-557.

5. HEDGING RESULTS FOR A PORTFOLIO OF ANNUITIES

In order to test the hedge effectiveness of the S-Swap and S-Cap, we considered a portfolio of 4,000 life annuities (N = 4,000), sold to women aged 65 years at the inception of the contracts. To keep our focus on mortality and survival rates, we used a constant flat interest rate of 4% for all maturities. Let $B_0(t)$ denote the discount factor from the future time *t* to the inception of the contract. At inception, the asset, A, is the present value of all future payments of $1 expected to be made at the end of each year to the annuitants. The present value of the (risk-adjusted) expected payments is given by the expression:

$$A = N * \Sigma_{i=1}^{T} B_0(i)\, \tilde{S}_{65}^{i}$$

If τ_k denotes the stochastic variable representing the year-end just before the death of the k^{th} annuitant, then the liability, L_k, for the k^{th} annuitant is the present value of the following string of payments.

$$L_k = \Sigma_{i=1}^{\tau_k} B_0(i)$$

Thus the present value of all payments made to the individuals of the cohort is

$$L = \Sigma_{k=1}^{N} L_k$$

As at the date of inception, the present value of the surplus, D, is given by the difference of the two

$$D = A - L$$

D represents the expected surplus of the unhedged annuity portfolio.

If we hedge the annuity portfolio with an S-Swap, we need to include the swap payoff (P_{Swap}) in the calculation of the surplus, D_{Swap}. The surplus expression becomes:

$$D_{Swap} = A - L + P_{Swap}$$

Similarly, for an S-Cap hedge, the surplus function is adjusted for the Cap payoff, P_{Cap}, and the premium cost, C_{Cap}. The expression for the surplus is:

$$D_{Cap} = A - L + P_{Cap} - C_{Cap}$$

Both instruments cover yearly maturities out to T = 20 years.

We then simulate 10,000 historical mortality/survival rate forecasts for the next 20 years in order to obtain a distribution of the surpluses. The following table shows the results of the simulations with and without the hedges. The results are expressed as the surplus per contract, i.e. D/N, D_{Swap}/N and D_{Cap}/N.

Portfolio	Average	Standard Deviation	Skewness	Kurtosis	VaR 99%	ES 99%
Unhedged	0.3472	0.3384	-0.0308	3.0500	-0.4572	-0.5752
Swap	0.2758	0.2468	-0.1060	3.0698	-0.3285	-0.4127
Cap	0.3037	0.2937	0.2427	3.1382	-0.3392	-0.4310

We observe that the expected value of the surplus decreases as we attempt to reduce the longevity risk by hedging the annuity portfolio. The surplus reduction is expected since the pricing of the hedge instruments incorporates a risk premium. More interestingly, we note a substantial reduction for all risk measures in the simulation. Of particular interest, is the reduction of the Value at Risk (VaR) and the Expected Shortfall (ES), which can be linked to capital charges. For example, assuming that capital is three times the VaR amount (a historical standard for market risk), we get a return on capital of 25% for the unhedged position, while returns on capital are 28% and 30% for the swap and cap hedged portfolio, respectively.

We could also think of hedging the portfolio with 30 year instruments instead, but the benefits dwindle as the population left after 20 and 30 years is on average 68% and 30% respectively so that the longevity risk left becomes idiosyncratic.

We then considered an annuity portfolio comprised of multi-cohort female annuitants of ages varying from 65 to 69 years old. Because future mortality improvements are correlated amongst the cohorts, we generated the random variables $\Delta W_x(t)$, for x = 65, 66, 67, 68 and 69, with their historical variance-covariance matrix. Moreover, it is the high correlation between these rates that allows us to hedge a multi-cohort annuity portfolio with fewer swap or cap contracts than we have cohorts.

The following table presents the results of our simulations with respect to a multi-cohort annuity portfolio, unhedged and hedged using S-Swaps or S-Caps. We varied the number of hedging instruments from 1 to 5 contracts using different age combinations and we reported the gains on the returns on capital in the two tables.

Swap Portfolio Hedging Results

Hedging Strategy	Average	Standard Deviation	Skewness	Kurtosis	VaR99	ES99	Return on Capital
Without Hedging	0.3112	0.2066	-0.0663	2.9833	-0.17	-0.2517	0.6102
S_{65}^{20}	0.2406	0.177	-0.0303	3.0687	-0.177	-0.2439	0.4531
$S_{65,67}^{20}$	0.2379	0.1572	-0.0919	3.038	-0.1332	-0.1956	0.5953
$S_{65,67,69}^{20}$	0.2344	0.148	-0.1056	3.0305	-0.1145	-0.1741	0.6824
$S_{67,68,69}^{20}$	0.2314	0.1454	-0.0859	3.0158	-0.1110	-0.1660	0.6949
$S_{65,66,67,68}^{20}$	0.2456	0.1533	-0.0865	3.0094	-0.1291	-0.1756	0.6342
$S_{65,66,67,68,69}^{20}$	0.2426	0.1469	-0.0930	2.9912	-0.1090	-0.1583	0.7420

Source: Université de Sherbrooke

Cap Portfolio Hedging Results

Hedging Strategy	Average	Standard Deviation	Skewness	Kurtosis	VaR99	ES99	Return on Capital
Without Hedging	0.3233	0.2167	-0.0165	3.0156	-0.187	-0.2606	0.5763
C_{65}^{20}	0.2761	0.1922	0.1006	3.1457	-0.1596	-0.2213	0.5766
$C_{65,67}^{20}$	0.2745	0.1856	0.165	3.1623	-0.1396	-0.1975	0.6554
$C_{65,67,69}^{20}$	0.2724	0.1816	0.1858	3.1914	-0.132	-0.1857	0.6879
$C_{67,68,69}^{20}$	0.2699	0.1798	0.1877	3.1589	-0.1295	-0.1792	0.6947
$C_{65,66,67,68}^{20}$	0.2729	0.1838	0.1927	3.1374	-0.1306	-0.1846	0.6965
$C_{65,66,67,68,69}^{20}$	0.2720	0.1812	0.2125	3.1565	-0.1255	-0.1757	0.7224

Source: Université de Sherbrooke

The results suggest that the annuity provider should use three contracts or more to hedge the longevity risk over the 20-year period. However, the incremental benefits seem to diminish as we add contracts beyond three. The longevity risk hedger should also consider transaction costs and basis risk when evaluating alternate solutions. Hence, further analyses are required on both realistic portfolios, to measure the population basis risk and incorporate risk objectives into the design of cost effective hedging solutions.

6. OTHER CONSIDERATIONS AND NEXT STEPS

In summary, we modeled the evolution of the central mortality rates for the Canadian population from 1921 to 2011 using three models. The models were calibrated to fit the mortality data over different age cohorts and a risk adjustment factor was incorporated to the financial models based on the pricing of longevity-linked securities.

The risk-adjusted financial models were used to price S-Forwards and Longevity Caplets. Finally, we tested the effectiveness of these instruments as hedges against longevity risk for a portfolio of (female) annuitants aged 65 (single cohort) and an annuity portfolio made of five age cohorts varying between 65 to 69 years of age. The results suggest that such longevity derivatives would be an effective solution to reduce longevity risks.

Although preliminary results are encouraging, it is important to deepen our understanding of the longevity risk dynamics and further test the applicability of the financial models described above and possibly of other models with suitable characteristics. Moreover, we should set the bases for the development of effective tools necessary to a viable life market. For example, a thriving life market will require the availability of survivor indexes on a timely basis in order to settle the payments on the transactions and also to mark-to-market these transactions. Furthermore, we need to demonstrate that selected mortality forecasts and pricing models are robust and that their construction has the necessary transparency to obtain the buy-in from financial market participants.

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